Maxwell-Dirac-Isomorphism. XIII

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On the history of wave mechanics.

About 60 years ago Schrödinger, in the derivation of his wave mechanics, at first followed de Broglie's concept according to which every particle is accompanied by a wave [1]: "Every time a material particle possesses, in the most general sense, an energy U in a frame of reference, then in this frame of reference there exists a periodic phenomenon of frequency ν, which is defined through the relation $U = h\nu$." – It should be noted here that for de Broglie there were always two things: the particle *and* the wave.

Then Schrödinger considered the Hamilton-analogy [2, 3], according to which a particle in a potential behaves as a light ray in a refracting medium: When a point of mass moves under the influence of conservative forces, which possess a potential energy Φ, then its trajectory is identical to the path of a ray of light in an inhomogeneous medium whose index of refraction is given by

$$N = c\,\frac{\sqrt{2m(U-\Phi)}}{U}. \tag{1}$$

Here it should be kept in mind that for Hamilton – in contrast to de Broglie – there is always only *one* thing, i.e., either the particle *or* the wave, either the mechanical system or the respective optical one.

For a concrete physical object, e.g., for a Kepler system, therefore, there are always two model versions available: On the one hand the mechanical Kepler system of two masses, which provide each other with a potential. On the other hand the correlated electromagnetic Kepler system of two light-fields, which provide each other with a refraction. Both models are equivalent through the connection (1).

The question of whether there exists another essential relation between the two Hamilton systems besides this connection has been answered by Ehrenfest [4, 5]: The mechanical system describes the motion of the center of energy of the light-system.

Schrödinger inserted the connection (1) into the classical equation for light

$$\left(\Delta - \frac{N^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0 \tag{2}$$

and thus obtained his wave mechanics. According to this derivation the Schrödinger equation is the differential equation of a light-system with the refraction (1). If one specializes to a Coulomb potential

$$\Phi = \frac{\text{const}}{r} \tag{3}$$

in the connection (1), then (2) describes two standing light fields with the hydrogen spectrum, whereas the respective mechanical part describes a Kepler system of two equal masses [5]. The two masses represent the centers of energy of the two light fields.

The light-quality of the Schrödinger function, however, becomes most convincingly evident through the isomorphism [6] between relativistic wave mechanics

$$\left[\gamma\cdot\nabla + i\frac{\omega}{c}\begin{pmatrix}\left(1-\dfrac{\Phi - m_0 c^2}{\hbar\omega}\right)\mathbb{1} & \mathbb{0}\\ \mathbb{0} & \left(1-\dfrac{\Phi + m_0 c^2}{\hbar\omega}\right)\mathbb{1}\end{pmatrix}\right]\Psi = 0 \tag{4}$$

and source-free electrodynamics

$$\left[\gamma\cdot\nabla + i\frac{\omega}{c}\begin{pmatrix}\varepsilon\,\mathbb{1} & \mathbb{0}\\ \mathbb{0} & \mu\,\mathbb{1}\end{pmatrix}\right]\Psi = 0. \tag{5}$$

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To the end of his days Schrödinger was convinced that the wave fields of his equation would describe physical realities. The then predominating conservative mechanistic views led to a gradually increasing unreality of the Schrödinger fields in the course of the interpreation controversy, and to the settlement on a particle-and-wave-model in the framework of the Copenhagen interpretation: the amplitude merely states where the particle could be. Schrödinger had to accept the dematerialization of his wave fields, especially since he himself had never been able to give a tenable interpretation. His hydrogen-interpretation, suggestive in view of the Rutherford-Bohr-model,

$$\psi\psi^* = \varrho \quad (\varrho\text{: "smeared charge" of the orbiting electron}) \tag{6}$$

proved to be untenable. At that time, in the thirties, it could not occur to anyone to regard the standing waves of the light-equation (5), or respectively (4), as standing light. There was the strong Rutherford model, seemingly secured to a convincing way by scattering experiments; and the Bohr model, which, especially through Sommerfeld's refinement, could meet almost all expectations.

Today the old pure-particle-models are, of course, just serving introductory means, whereas in research and development almost exclusively Born's wave-and-particle-model is used. Its successes testify to the great thruthfulness of that picture. Nevertheless, in view of the Maxwell-Dirac-isomorphism, today we have to ask ourselves: Had Schrödinger been right? Do his fields possess reality? Do we have to comprehend hydrogen as standing light?

[1] L. de Broglie, These de doctorat. J. de Physique **1,** 1 (1926).
[2] W. R. Hamilton, Trans. Roy. Irish Acad. **15,** 69 (1828); **16,** 4 and 93 (1830); **17,** 1 (1837).
[3] W. R. Hamilton, Math. Papers **1,** 484 (1837).
[4] P. Ehrenfest, Z. Physik **45,** 455 (1927).
[5] H. Sallhofer, Z. Naturforsch. **35 a,** 995 (1980).
[6] H. Sallhofer, Z. Naturforsch. **41 a,** 468 (1986) and Corrigenda in: H. Sallhofer, Z. Naturforsch. **41 a,** 1087 (1986).

Erratum

K. Murawski, A Note on Solutions of the Korteweg-de Vries Equation, Z. Naturforsch. **40 a,** 193–194 (1985).

The formula (17) should be written as follows

$$p_2 = g\,k^2 \operatorname{sech}^2(g\,k\,\eta) + 4\,g\,k^2 \frac{4\,k\,C_1 \operatorname{sh}(2\,g\,k\,\eta) - \operatorname{ch}(2\,g\,\eta)}{[8\,k\,C_1 \operatorname{ch}^2(g\,k\,\eta) - \operatorname{sh}(2\,g\,k\,\eta)]^2}, \tag{17 a}$$

$$p_2 = -g\,k^2 \operatorname{csch}^2(g\,k\,\eta) + 4\,g\,k^2 \frac{4\,k\,C_1 \operatorname{sh}(2\,g\,k\,\eta) + \operatorname{ch}(2\,g\,k\,\eta)}{[8\,k\,C_1 \operatorname{sh}^2(g\,k\,\eta) + \operatorname{sh}(2\,g\,k\,\eta)]^2}. \tag{17 b}$$

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